There exists an integer, p, called the degree of L, such that $Lu(x) = O(h^{p+r})$ as $h \to 0$. L is said to be stable if all zeros of $\rho(\zeta)$ are of modulus $\leqq 1$, and the zeros with modulus 1 are of multiplicity $\leqq r$.

The author shows that stability of L is equivalent to "stable convergence" of certain solutions of the difference equation associated with L to the solution of equation (2). If L is unstable the numerical solutions are "strongly unstable," and the integration formula is practically useless. Several theorems are given concerning the largest possible degree of stable operators of a given order. For example, if $\tau(\zeta) \equiv 0$, L is unstable if $p > 2([k/2] + [(r + 1)/2])$; here $[x]$ denotes the largest integer $\leqq x$.

If the initial-value problem, $dy/dx = qy$, $y(0) = 1$, q constant, is treated numerically with a stable operator L of order k and degree $k + 2$, where $\tau(\zeta) \equiv 0$, the solution is a linear combination of basic solutions, ζ_{jh}^n, $1 \leqq j \leqq k$, $n = 0, 1, 2, \cdots$, where ζ_{jh} are the roots of the characteristic polynomial, $\rho(\zeta) - qh\sigma(\zeta)$. Let ζ_j be the roots of $\rho(\zeta)$; in the present case, $|\zeta_j| = 1$ and ζ_j is single for all j. The author shows that, for $h \to 0$, $\zeta_{jh} \sim \zeta_j(1 + k_j qh)$, where the $k_j = \sigma(\zeta_j)/[\zeta_j \rho'(\zeta_j)]$ are called growth parameters and are real numbers, then $\zeta_{jh}^n \sim \zeta_j^n \exp(k_j qhn)$. One of the ζ_j, say ζ_1, is equal to 1, and $k_1 = 1$.

If $Re(q) < 0$, the solution $\zeta_{1,h}^n \sim e^{qhn}$ decays, but may be dominated by some of the other basic solutions which increase or decrease more slowly. If there exist solutions which increase for $Re(q) < 0$, the operator L is called weakly unstable; this occurs if at least one of the k_j, $j \geqq 2$, is <0. It is shown that a stable operator of even order k and maximum degree $k + 2$ is weakly unstable; such an operator generates an oscillatory solution whose amplitude increases at least as rapidly as $|\exp(-qnh/3)|$ if $Re(q) < 0$.

In the second part of this thesis the author is concerned with estimating the norm of the error vector for $r = 1$. First, the error is evaluated for the linear variational system $d\bar{z}/dx = B(x)\bar{z}$ associated with (2), where $B(x)$ is the Jacobian, $(\partial\bar{f}/\partial\bar{y})_{\bar{y}=\bar{y}(x)}$. Then the effect of the linearization is estimated separately. Three error formulas are given. The second one, involving the directional derivative, $\mu[B(x)] = \lim_{\lambda \to 0+} \lambda^{-1}[\| I + \lambda B(x) \| - 1]$, yields particularly good results if the numerical solution is smooth and if $\mu[B(x)] < 0$.

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81[X].—A KORGANOFF, with the collaboration of L. BOSSETT, J. L. GROBOILLOT & J. JOHNSON, *Méthodes de Calcul Numérique*, Tome I: *Algèbre non linéaire*, Dunod, Paris, 1961, xxvii + 375 p., 25 cm. Price 58 NF.

The volume being reviewed is the first of a projected series. The characteristic-value problem is included, but matrix inversion is not, except to the extent that an iterative method that applies to a system of nonlinear equations would apply also to a system of linear equations.

The French literature on numerical analysis is sparse indeed, and hitherto has been but slightly affected by the advent of the electronic computer. This book goes far toward filling the gap there, and would be a substantial contribution to the literature in any language.

A short initial chapter discusses digital computers and programming in rather general terms. Chapter 2, "Erreurs," consisting of 70 pages, is perhaps the most complete and systematic single account in existence, discussing both rigorous bounds and probabilistic estimates. Chapter 3 considers iterative methods in general as applied to transcendental equations and to systems of equations. Chapter 4 is devoted to polynomials specifically, and Chapter 5 to characteristic values. Each chapter is supplemented by an extensive bibliography. An appendix gives numerical results of applying the various methods to some specific problems.

One can find fault with a few details. The method of Graeffe is given in the chapter on polynomial equations, although it is equally applicable to transcendental equations within a circle of analyticity. In the chapter on the characteristic-value problem, there is an interesting general section which derives the Jordan normal form, and develops the standard localization theorems of Gershgorin, Brauer, and others. But the treatment of the methods is somewhat disappointing, especially of the so-called "direct methods" (actually methods of reduction, although the reviewer himself is guilty of having propagated the misnomer). It is even stated that the method of Krylov is not to be recommended for large matrices, whereas it has been shown by Bauer and this reviewer that most of the known direct methods are only specializations of the Krylov method.

However, the authors could justify the rather sketchy treatment of the methods of reduction by arguing that these do not differ basically from methods of inversion, and are hence peripheral to their main interest, which would be the solution of the polynomial equation that can be obtained from a complete reduction. The treatment of this problem is by no means sketchy. In fact, it is the most complete and up-to-date account of known methods now in the literature, exhibiting well their interrelations, and often presenting them from a fresh point of view. This volume can be recommended highly, and one can hope that the subsequent ones will be equally well done.

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82[X].—Herbert E. Salzer, Dexter C. Shoultz, & Elizabeth P. Thompson, *Tables of Osculatory Integration Coefficients*, Convair (Astronautics) Division of General Dynamics Corporation, San Diego, 1960, 43 p., 28 cm. (Paperback)

This publication contains tables to implement the use of quadrature formulas of the so-called osculating type, i.e., formulas in which the value of the integrand function and of its derivative at each point are used. The explicit formula considered is

$$(1) \qquad \int_{x_0+qh}^{x_0+ph} f(u)\,du = h \sum_{i=-[(n-1)/2]}^{[n/2]} \{C_i^{(n)}(p)f_i + D_i^{(n)}(p)hf_i'\} + R_{2n}(p).$$

The authors treat $n = 2, 3, 4, 5$; for $n = 2$ and 4, q is taken to be $\frac{1}{2}$, and for $n = 3$ and 5, q is taken to be 0. These choices of q permit use of symmetry relations to reduce the amount of tabulation. The coefficients $C_i^{(n)}(p)$ and $D_i^{(n)}(p)$ then are polynomials in p, and these are listed for each i indicated in (1). Ten-decimal tables